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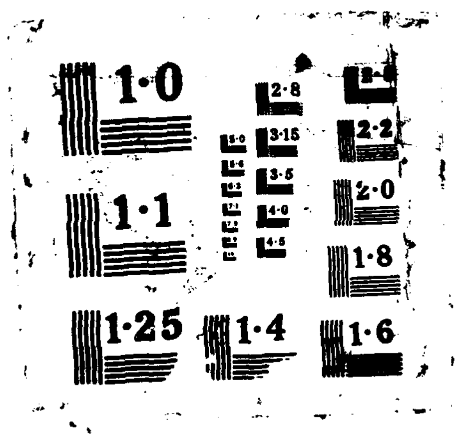
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~~We~~ This paper considers prediction for stationary processes, in environments where data outliers may be present. ~~We~~ The develops a sequence of outlier resistant prediction operations, which is uniformly qualitatively robust. ~~We study~~ Studied are the asymptotic mean-squared performance of the developed operations, both in the absence and the presence of i.i.d. data outliers. Important performance characteristics studied include the breakdown point and the influence function.

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I. INTRODUCTION

In prediction, data generated by some stochastic process are deduced from past observations. Given a well-known such process, the optimal mean-squared predictor is the conditional mean, which is generally a complicated function of the past observations. Linear prediction operations are then widely used, due to their simplicity, and the classical theory of linear prediction for weakly stationary discrete-time processes is mainly due to Wiener, [19], and Kolmogorov, [11],[12]. However, such linear operations are notoriously unstable in the presence of contaminations due to data outliers, (see Huder, [9], and Hampel, [7]), while the occurence of such outliers is a phenomenon frequently observed in practice. In this paper, we develop and analyze a sequence of outlier resistant prediction operations. Our presentation combines the theories of saddle point games and qualitative robustness, (for the latter see Boente et al, [1], Cox [2], Hampel, [7], Papantoni-Kazakos and Gray [13], and Papantoni-Kazakos, [14], [15], [16],). Similar approach was used by Tsaknakis, [18], for the development of outlier resistant filtering and smoothing operations.

Considerable effort has been dedicated to the development of minimax linear prodictors, in cases when the spectral density of the process is not well defined, but is instead a member of some compact class, (see Franke, [3], Franke and Poor, [4], Hosoya, [8], Kassam and Poor, [10], and Tsaknakis et al, [17]). Such predictors are highly sensitive to data outliers, however.

In this paper, one-step prediction is considered, and the organization is as follows: In section II, we present formalization of the problem and we define important performance criteria for outlier resistant operations. In section III, we develop outlier resistant prediction operations and we study their asymptotic performance. In section IV, we examine the special case of first order autoregressive nominal processes. In section V, we draw some conclusions.

II. PRELIMINARIES

Let R be the real line, and let $\mathcal{B}$ be the usual Borel σ-field on R. Let R^{∞} be the one-sided sequence space, and let $\mathcal{B}^{\infty}$ be the Borel σ-field on R^{∞} that is generated by the product topology on R^{∞}. We consider a real-valued discrete-time process, $\{X_n, 1\leq n<\infty\}$, whose measure μ_o is known and is defined on $\mathcal{B}^{\infty}$. We name $\{X_n, 1\leq n<\infty\}$ the <u>nominal process</u>, and we denote by $\{x_n, 1\leq n<\infty\}$ data realizations generated by it. Let $\hat{x}_n = \hat{x}_n(x_1^{n-1})$ denote the optimal one-step mean-squared prediction operation, given the sequence realization $x_1^{n-1} \triangleq \{x_\ell, 1\leq \ell\leq n-1\}$, when $\{x_n, 1\leq n<\infty\}$ is generated by the nominal process. Then if $g_n = g_n(x_1^{n-1})$ denotes some scalar real-valued function on the sequence x_1^{n-1}, we have:

$$e_n(\mu_o,\hat{x}_n) = \inf_{g_n} e_n(\mu_o,g_n) \tag{1}$$

$$\hat{x}_n(x_1^{n-1}) = E_{\mu_o}\{X_n|x_1^{n-1}\} \tag{2}$$

; where $E_{\mu_o}\{\ \}$ denotes expectation with respect to the measure μ_o, where $X_1^n \triangleq \{X_\ell, 1\leq \ell\leq n\}$, and where,

$$e_n(\mu_o,g_n) \triangleq E_{\mu_o}\{[X_n-g_n(X_1^{n-1})]^2\} \tag{3}$$

The expression in (3) is called the one-step prediction error induced by g_n at μ_o. Let L_n denote the class of all the scalar real-valued linear functions defined on R^n. Let then $\hat{x}_n^L = \hat{x}_n^L(x_1^{n-1})$ be such that:

$$e_n(\mu_o,\hat{x}_n^L) = \inf_{g_n^L\in L_{n-1}} e_n(\mu_o,g_n^L) \tag{4}$$

Then, $\hat{x}_n^L$ is called the optimal linear one-step mean squared predictor at μ_o, given the sequence realization x_1^{n-1}, and generally,

$$e_n(\mu_o,\hat{x}_n) \le e_n(\mu_o,\hat{x}_n^L) \tag{5}$$

If the measure μ_o is Gaussian, then $\hat{x}_n(x_1^{n-1}) \triangleq \hat{x}_n^L(x_1^{n-1})$, $\forall n$, and (5) is then satisfied with equality for all n. If μ_o is non Gaussian, then (5) is generally a strict inequality.

The above summary corresponds to parametric one-step prediction; that is, it corresponds to the case where the measure μ_o that generates the data sequences is known. In this paper, we are concerned with the outlier model. Then, the observation process $\{Y_n, 1\le n<\infty\}$ is generated by three mutually independent processes, the nominal process $\{X_n, 1\le n<\infty\}$ and two i.i.d. processes $\{V_n, 1\le n<\infty\}$ and $\{Z_n, 1\le n<\infty\}$, as follows:

$$Y_n = (1-V_n)X_n + V_n Z_n \quad , n=1,2,... \tag{6}$$

; where the common distribution of the variables Z_n, $1\le n$, is unknown, and where $\{V_n, 1\le n<\infty\}$ is a binary process. In particular, for some given ε: $0\le\varepsilon<1$, the latter process is such that:

$$\begin{aligned} P(V_k = 0) &= 1-\varepsilon \\ P(V_k = 1) &= \varepsilon \end{aligned} \tag{7}$$

In the outlier model in (6), $\{Z_n, 1\le n<\infty\}$ is called the contaminating process, and $\{V_n, 1\le n<\infty\}$ determines the contamination law. In the presence of the latter model, the objective is predicition of the nominal datum x_n, given the observation sequence y_1^{n-1}, for all n, and the problem formalization is then clearly non-parametric. Let μ denote the measure of the observation process, and let $\{g_n\}_{1\le n<\infty}$ denote a sequence of one-step predictors, where $g_n = g_n(y_1^{n-1})$. Let us then define,

$$e_n(\mu,g_n) \triangleq E_\mu\{[X_n-g_n(Y_1^{n-1})]^2\} \tag{8}$$

In (8), $e_n(\mu,g_n)$ is the mean-squared error induced by the predictor g_n, when the measure of the observation process $\{Y_n,\ 1\leq n<\infty\}$ is μ, and where X_n is generated by the nominal process whose measure is μ_o. Clearly, $e_n(\mu_o,g_n)$ is then as in (3), and it represents the mean-squared performance of the predictor g_n at the nominal measure μ_o, (that is, when outliers are absent).

Our objective is to design a sequence $\{g_n\}_{1\leq n<\infty}$ of predictors whose mean-squared performance is stable in the presence of variations in the measure μ of the observation process $\{Y_n,\ 1\leq n<\infty\}$. This stability corresponds to qualitative robustness, and is defined as follows:

Given $\eta>0$, there exists $\delta>0$, such that:

$$\Pi_\rho(\mu_o,\mu) <\delta \quad \text{implies} \quad |e_n(\mu_o,g_n) - e_n(\mu,g_n)|<\eta \ ; \ \forall n$$

In the above definition, Π_ρ denotes Prohorov distance with an appropriate distortion measure ρ on data sequences, and sequences $\{g_n\}$ of operations that satisfy this stability are called <u>qualitatively robust</u> at the measure μ_o. As found first in [13], and later in [1], [14], and [16], for the sequence $\{g_n\}$ to be qualitatively robust, pointwise continuity and asymptotic continuity in conjuction with boundness, are sufficient. In particular, it is sufficient that g_n is bounded for all n, and:

(A) Given finite n, given $\eta>0$, given x_1^n, there exists $\delta>0$, such that, $y_1^n : \gamma_n(x_1^n,y_1^n) \triangleq n^{-1}\sum_{i=1}^{n}|x_i-y_i|<\delta$ implies $|g_{n+1}(x^n_.)-g_{n+1}(y_1^n)|<\eta$.

(B) Given μ_o stationary, given $\zeta>0$, $\eta>0$, there exist integers n_o, m, some $\delta>0$, and for each $n>n_o$ some $\Delta^n\varepsilon R^n$ with $\mu_o(\Delta^n)>1-\eta$, such that for each $x^n\varepsilon\Delta^n$ and y^n such that $\inf\{\alpha: \#[i : \gamma_m(x_i^{i+m-1}, y_i^{i+m-1})>\alpha]\leq n\alpha\}<\delta$ it is implied that $|g_{n+1}(x_1^n)-g_{n+1}(y_1^n)|<\zeta$.

Given a sequence $\{g_n\}$ of predictors which is qualitatively robust at the nominal measure μ_o, its important quantitative performance criteria are: (1) Its asymptotic mean-squared performance at the nominal measure, $\limsup_{n\to\infty} e_n(\mu_o, g_n)$ (2) Its breakdown point. (3) Its influence function. The breakdown point and the influence function represent measures of resistance to outliers, and their definitions are given below.

Consider the model in (6), and let then $\{Z_n\}$ be a deterministic process with amplitude w; that is, $P(Z_n = w) = 1$. Let then $\mu_{\varepsilon,w}$ be the measure of the observation process $\{Y_n\}$. Given a sequence $\{g_n\}$ of predictors, we then define:

<u>Influence Function</u> of the sequence $\{g_n\}$:

$$I_g(w) \stackrel{\Delta}{=} \lim_{\varepsilon\to 0} \frac{e(\mu_{\varepsilon,w}, g) - e(\mu_o, g)}{\varepsilon} \tag{9}$$

; where,

$$e(\mu, g) \stackrel{\Delta}{=} \limsup_{n\to\infty} e_n(\mu, g_n) \tag{10}$$

<u>Breakdown point</u> of the sequence $\{g_n\}$:

$$\varepsilon_g^* \stackrel{\Delta}{=} \sup \{ \varepsilon : e(\mu_{\varepsilon,\infty}, g) \leq \limsup_{n\to\infty} E_{\mu_o}\{X_n^2\}\} \tag{11}$$

; where $e(\mu, g)$ is defined as in (10).

We note that the breakdown point is the maximum frequency of independent, infinite-amplitude outliers that the prediction sequence can tolerate asymptotically, without becoming useless, (that is, before the observation sequences provide no information about the next process datum). The influence function represents the slope of the function $e(\mu_{\varepsilon,w}, g) - e(\mu_o, g) \stackrel{\Delta}{=} F_{\varepsilon,g}(w)$, at the $\varepsilon = 0$ point. $F_{\varepsilon,g}(w)$ corresponds to the asymptotic mean-squared error increase induced by the prediction sequence $\{g_n\}$, when from absence of outliers the environment shifts to ε-frequency and w-amplitude outlier occurence.

The outlier model in (6) can be generalized to i.i.d. sequences of m-size blocks of outliers, as follows:

$$Y^{km}_{(k-1)m+1} = (1-V_k)X^{km}_{(k-1)m+1} + Z^{km}_{(k-1)m+1} \quad ; \; k=1,2,\ldots \tag{12}$$

; where the sequence $\{V_n\}$ is as in (7), and where the vector random variables $\{Z^{km}_{(k-1)m+1}\}$ are i.i.d. with unknown distribution. Let $\mu_{\varepsilon,w,m}$ denote the measure of the observation process $\{Y_n\}$, when the model in (12) is present, and when $P(Z_n=w)=1$. Then, given a sequence $\{g_n\}$ of predictors, and defining $e(\mu,g)$ as in (10), the breakdown point, $\varepsilon^*_{g,m}$, and the influence function, $I_{g,m}(w)$, that correspond to the outlier model in (12) are defined as follows:

$$\varepsilon^*_{g,m} \triangleq \sup \{ \varepsilon: e(\mu_{\varepsilon,\infty,m},g) \leq \lim_{n\to\infty} \sup E_{\mu_o}\{X_n^2\}\} \tag{13}$$

$$I_{g,m}(w) \triangleq \lim_{\varepsilon\to 0} \frac{e(\mu_{\varepsilon,w,m},g) - e(\mu_o,g)}{\varepsilon} \tag{14}$$

III. OUTLIER RESISTANT PREDICTION OPERATIONS

We consider a stationary, zero mean, real-valued process $\{X_n, 1\leq n<\infty\}$, with measure μ_o, and $E_{\mu_o}\{X_n^2\} = \sigma^2<\infty$. We also consider the outlier model in (12) for the observation process $\{Y_n, 1\leq n<\infty\}$. We concentrate on the design of qualitatively robust and outlier resistant sequences $\{g_n\}$ of one-step predictors for the process $\{X_n, 1\leq n<\infty\}$. Our methodology involves two steps: (1) A saddle-point game formalization and solution for the predictors g_n : $2\leq n\leq m+1$. (2) A qualitatively robust generalization of the solutions in step 1, for the predictors g_n : $n>m+1$.

In the sequel, we will assume that both the nominal and the contaminating processes are absolutely continuous. We will then denote by $f_o(x_1^n)$ the density function induced by the nominal process at the vector point x_1^n; we will denote by $f(y_1^n)$ the density function induced by the observation process at the vector point y_1^n. We note that then, for n : $2\leq n\leq m+1$, the class, $\mathcal{F}_n$, of density functions induced by the model in (12) is as follows:

$$F_n=\{f:f(y_1^{n-1})-(1-\varepsilon)f_o(y_1^{n-1}) \geq 0;\forall\ y_1^{n-1}\varepsilon R^{n-1},\quad \int_{R^{n-1}} f(y_1^{n-1})dy_1^{n-1}=1\} \tag{15}$$

Construction of Prediction Operations - Step 1

Let us consider the model in (12) and one-step prediction based on observation sequences y_1^{n-1}, with $n : 2\leq n\leq m+1$. Given such an n, we consider the following saddle point game, where F_n is as in (15):

Find a pair, (f^*,g_n^*), of an observation density function and an one-step predictor, such that $f^*\varepsilon F_n$, and:

$$\forall f\varepsilon F_n \quad ; \quad e_n(f,g_n^*) \leq e_n(f^*,g_n^*) \leq e_n(f^*,g_n) \quad ; \forall g_n \tag{16}$$

In (16), the errors $e_n(f,g_n)$ are as in (8), where the measure, μ, has been substituted by the corresponding density function, f.

Consider a pair, (f',g_n'), of an observation density and a prediction operation, such that:

$$(f',g_n') \ : \ e_n(f',g_n')= \sup_{f\varepsilon F_n} \inf_{g_n} e_n(f,g_n) \tag{17}$$

From the results in [15] we then conclude that if the operation $g_n'=g_n'(y_1^{n-1})$ is pointwise continuous and bounded, then $(f',g_n') = (f^*,g_n^*)$, and the pair is a unique solution of the game in (16). We now present a theorem whose proof is in the Appendix.

Theorem 1

Let the nominal process be zero mean Gaussian. Let then P_n denote the n-dimensional autocovariance matrix of this process, and let $m_o(y_1^{n-1}) = B_{n-1}^T y_1^{n-1}$ denote the optimal at the Gaussian nominal process one-step predictor, when the observation sequence is y_1^{n-1}. Let $n:2\leq n\leq m+1$. Then, the pair (f',g_n') in (17) is as follows:

$$g_n'(y_1^{n-1}) = m_o(y_1^{n-1})\cdot \min(1,\lambda_n\{(y_1^{n-1})^T P_{n-1}^{-1} y_1^{n-1}\}^{-1/2}) \tag{18}$$

$$f'(y_1^{n-1}) = (1-\varepsilon) f_o(y_1^{n-1})\cdot \max(1,\lambda_n^{-1}\{(y_1^{n-1})^T P_{n-1}^{-1} y_1^{n-1}\}^{1/2}) \tag{19}$$

; where λ_n is unique, and such that:

$$\lambda_n : \int_{R^{n-1}} f'(y_1^{n-1}) dy_1^{n-1} = 1 \tag{20}$$

Since the operation in (18) is pointwise continuous and bounded, $(f',g_n') \equiv (f^*,g_n^*)$, and the pair is a unique solution of the game in (16).

When the nominal process is non Gaussian, the operation g_n' in (17) is generally not pointwise continuous; thus, there is no guarantee then that it will satisfy the game in (16), and it is generally qualitatively nonrobust. However, drawing from linear prediction in the absence of outliers, we will adopt the operations in Theorem 1, for non Gaussian nominal processes as well. Specifically:

Let the nominal process be stationary and zero mean, with n-dimensional autocovariance matrix P_n. Let $m_o(y_1^{n-1}) = B_{n-1}^T y_1^{n-1}$ denote the optimal at the nominal process linear one-step predictor when the observation sequence is y_1^{n-1}. Let f_G denote densities of the Gaussian process whose power spectral density is the same as that of the nominal process, and whose mean is zero. Then, in the presence of the outlier model in (12), and for $n : 2 \le n \le m+1$, we adopt the following one-step prediction operation:

$$g_n^*(y_1^{n-1}) = m_o(y_1^{n-1}) \quad \min(1,\lambda_n\{(y_1^{n-1})^T P_{n-1}^{-1} y_1^{n-1}\}^{-1/2})$$

$$\lambda_n : \int_{R^{n-1}} f_G(y_1^{n-1})\cdot \max(1,\lambda_n^{-1}\{(y_1^{n-1})^T P_{n-1}^{-1} y_1^{n-1}\}^{1/2}) = (1-\varepsilon)^{-1} \tag{21}$$

We note that for $\varepsilon=0$, the value of λ_n is infinity and the operation g_n^* becomes identical to the optimal at the nominal linear one-step predictor. As ε increases, λ_n decreases monotonically, becoming zero at $\varepsilon=1$.

Construction of Prediction Operations - Step 2

In this part, we are concerned with the construction of qualitatively robust prediction operations, for large dimensionalities of observation sequences. We point out that the operations in (21) are qualitatively robust for finite such dimensionalities only. Indeed, they satisfy condition (A) in section II and are bounded, but they do not satisfy condition (B). At the same time, the outlier model in (12) does not allow for the formalization of a saddle point game for arbitrary data dimensionalities, even when the nominal process is Gaussian. We will thus adopt an adhoc approach.

Let $\{a_j^{(n)}\}_{1\le j\le n}$ denote the one-step prediction coefficients of the nominal process, when n observation data are available. That is, if $m_o(y_1^n)$ denotes the optimal at the nominal linear one-step predictor when the observation sequence is y_1^n, then:

$$m_o(y_1^n) = \sum_{j=1}^{n} a_j^{(n)} y_j \tag{22}$$

Let g_n^* be as in (21). Then, we propose the following sequence, $\{G_n^*\}$, of one-step predictors:

$$G_n^*(y_1^{n-1}) = g_n^*(y_1^{n-1}) \quad ; \quad \text{for } 2\le n\le m+1.$$

$$\begin{aligned} G_n^*(y_1^{n-1}) &= \sum_{j=1}^{m} a_j^{(n-1)} \frac{g_{j+1}^*(y_1^j)-g_{j+1}^*(0,y_1^{j-1})}{a_j^{(j)}} \\ &+ \sum_{j=m+1}^{n-1} a_j^{(n-1)} \frac{g_{m+1}^*(y_{j-m+1}^j)-g_{m+1}^*(0,y_{j-m+1}^{j-1})}{a_m^{(m)}} \quad ; \text{ for } n>m+1 \end{aligned} \tag{23}$$

; where $(0,y_\ell^{\ell+n})$ denotes the sequence $\{y_\ell,y_{\ell+1},\ldots,y_{\ell+n},0\}$.

We observe that the sequence $\{G_n^*\}$ in (23) degenerates to the sequence of the optimal at the nominal linear predictors, when in the model in (12), $\varepsilon=0$,

(design in the absence of outliers). In addition, using a similar proof as in [15], we can easily show that the sequence $\{G_n^*\}$ is qualitatively robust, (satisfying condition (B) in section II), if:

$$\sup_k \sum_{j=1}^{k} |a_j^{(k)}| \le c^* < \infty \tag{24}$$

<u>Asymptotic Performance at the Nominal Process</u>

In this part, we focus on the asymptotic mean-squared error induced by the sequence $\{G_n^*\}$ in (23), at the nominal process. In particular, we wish to evaluate $e(\mu_o, G)$, where,

$$e(\mu_o, G^*) = \limsup_{n\to\infty} e_n(\mu_o, G_n^*) \tag{25}$$

Let e_o denote the asymptotic mean-squared error induced by the optimal at the nominal linear mean-squared predictor, when the observations are generated by the nominal process; that is,

$$e_o \triangleq \limsup_{n\to\infty} E_{\mu_o}\{[X_{n+1} - \sum_{j=1}^{n} a_j^{(n)} Y_j]^2\} \tag{26}$$

Let us also define,

$$d^* \triangleq \limsup_{k\to\infty} \sum_{j=1}^{k} |a_j^{(k)}| \tag{27}$$

$$D_m^* \triangleq E_{\mu_o}^{1/2}\left\{\left[Y_m - \frac{g_{m+1}^*(Y_1^m) - g_{m+1}^*(0, Y_1^{m-1})}{a_m^{(m)}}\right]^2\right\} \tag{28}$$

Then, we can express the following theorem, whose proof is in the Appendix.

<u>Theorem 2</u>

Let the nominal process be zero mean and stationary, with $d^* < \infty$, $\limsup_{k\to\infty} \sum_{j=1}^{m} |a_j^{(k)}| = 0$, and $E_{\mu_o}\{X^2\} < \infty$. Then,

$$e(\mu_o, G^*) \le E_{\mu_o}\{X^2\} \tag{29}$$

$$|e^{1/2}(\mu_o, G^*) - e_o^{1/2}| \le d^* D_m^* \tag{30}$$

We note that D_m^* decreases to zero, when the parameter λ_m in (21) goes to infinity. Then, the asymptotic mean-squared error, $e(\mu_o, G^*)$, becomes identical to the optimal at the nominal linear mean-squared error, e_o. Also, D_m^* is bounded from above by $E_{\mu_o}^{1/2}\{Y_m^2\}$, for every λ_m value.

Outlier Resistance

In this part, we are focusing on the properties of the breakdown point and the influence function induced by the one-step prediction sequence $\{G_n^*\}$ in (23). We note that, as well known, the breakdown point of the optimal at the nominal linear one-step prediction sequence is zero, and its influence function, $I(w)$, is quadratic; thus unbounded, (see [18]). We now state the following theorem, whose proof is in the Appendix.

Theorem 3

Let the nominal process be stationary and zero mean, with $d^* \triangleq \limsup_{k\to\infty} \sum_{j=1}^{k} |a_j^{(k)}|<\infty$, $E_{\mu_o}\{X^2\}<\infty$, and $\limsup_{k\to\infty} \sum_{j=1}^{m} |a_j^{(k)}| =0$, for every given finite m. Let λ_m in (21) be bounded. Then, the sequence $\{G_n^*\}$ in (23) has strictly positive breakdown point, and bounded influence function.

IV. GAUSSIAN AUTOREGRESSIVE NOMINAL PROCESS

In this section, we consider a first-order autoregressive and Gaussian nominal process, and we study then the performance of the sequence, $\{G_n^*\}$, in (23), in detail. In particular, let the nominal process $\{X_n, 1\le n<\infty\}$ be such that:

$$X_n = \alpha X_{n-1} + W_n \tag{31}$$

; where $\alpha<0.5$ and where the variables $\{W_n\}$ are i.i.d. and zero-mean, unit-variance Gaussian. The process $\{X_n, 1\le n\le\infty\}$ is then zero mean, and asymptotically stationary with $\limsup_{n\to\infty} E_{\mu_o}\{X_n^2\} = (1-\alpha^2)^{-1}$.

Considering the above nominal process, Theorem 1 applies, and the operations $\{g_n'\} = \{g_n^*\}$ in (18) and (23) take here the following asymptotic form:

For $j\to\infty$, $g^{*}_{m+1}(y^{j}_{j-m+1}) = \alpha y_j \cdot \min(1, \lambda_m \{y^2_{j-m+1} + (1-\alpha^2) \sum_{i=j-m+2}^{j} (y_i - \alpha y_{i-1})^2\}^{-1/2})$ (32)

; where,

$$\lambda_m : \int_{R^m} \max(1, \lambda_m^{-1} \{y_1^2 + (1-\alpha^2) \sum_{i=2}^{m} (y_i - \alpha y_{i-1})^2\}^{1/2}) \cdot (2\pi)^{-\frac{m}{2}} (1-\alpha^2)^{-\frac{1}{2}}$$

$$\cdot \exp\{- \frac{1}{2(1-\alpha^2)} [y_1^2 + (1-\alpha^2) \sum_{i=2}^{m} (y_i - \alpha y_{i-1})^2]\} dy_1^m = (1-\varepsilon)^{-1} \tag{33}$$

For $n\to\infty$, $G^{*}_n(y_1^{n-1}) = g^{*}_{m+1}(y^{n-1}_{n-m})$ (34)

From the above expressions, we easily find the following expressions, where $\Phi(x)$ and $\phi(x)$ denote respectively the distribution and the density functions of the zero-mean and unit-variance Gaussian variable, at the point x.

For $j\to\infty$, $g^{*}_2(y_j) = \alpha y_j \min(1, \frac{\lambda_1}{|y_j|})$ (35)

$$\lambda_1 : 2\Phi(\frac{\lambda_1}{\sqrt{1-\alpha^2}}) - 1 + 2\frac{\sqrt{1-\alpha^2}}{\lambda_1}\phi(\frac{\lambda_1}{\sqrt{1-\alpha^2}}) = (1-\varepsilon)^{-1} \tag{36}$$

For $j\to\infty$, $g^{*}_3(y_j, y_{j-1}) = \alpha y_j \min(1, \lambda_2\{y^2_{j-1} + (1-\alpha^2)(y_j - \alpha y_{j-1})^2\}^{-1/2})$ (37)

$$\lambda_2 : \sqrt{\frac{\pi}{2}} \left\{ 2\Phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}}) - 1 + 2\frac{\sqrt{1-\alpha^2}}{\lambda_2}\phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}}) \right\} = (1-\varepsilon)^{-1} \tag{38}$$

The functions that determine the λ_1 and λ_2 values, in (36) and (38) respectively, are both motonically decreasing with increasing λ; from ∞ to 1; thus, for $\varepsilon<1$, both λ_1 and λ_2 are unique. In addition, it can be easily seen that $\lambda_2 > \lambda_1$.

We will study the operations in (34), for m=1 and m=2. That is, we will analyze the operations in (35) and (37), in terms of performance at the nominal, breakdown point, and influence function.

<u>Case m=1</u>

Then, from (35) and (34) and for λ_1 as in (36), we obtain:

$$\text{For } n\to\infty,\ G_n^*(y_1^{n-1}) = \alpha y_{n-1}\cdot\min\left(1, \frac{\lambda_1}{|y_{n-1}|}\right) \tag{39}$$

Then, we easily find:

$$\begin{aligned} e(\mu_o,G^*) &= \limsup_{n\to\infty} \left(E_{\mu_o}\{X_n^2\} - 2E_{\mu_o}\{X_n G_n^*(Y_1^{n-1})\} + E_{\mu_o}\{[G_n^*(Y_1^{n-1})]^2\}\right) \\ &= (1-\alpha^2)^{-1}\{1-\alpha^2[2\Phi(\lambda_1\sqrt{1-\alpha^2}) -1]+2\alpha^2(1-\alpha^2)\lambda_1^2[1-\Phi(\lambda_1\sqrt{1-\alpha^2})] \\ &\quad - 2\alpha^2\sqrt{1-\alpha^2}\,\lambda_1\,\phi(\lambda_1\sqrt{1-\alpha^2})\} \end{aligned} \tag{40}$$

$$\begin{aligned} I_{G^*}(w) &= \alpha^2 w^2\cdot\min\left(1, \frac{\lambda_1^2}{w^2}\right) + (1-\alpha^2)^{-1}\alpha^2\{[2\Phi(\lambda_1\sqrt{1-\alpha^2})-1] - \\ &\quad - 2(1-\alpha^2)\lambda_1^2[1-\Phi(\lambda_1\sqrt{1-\alpha^2})] + 2\sqrt{1-\alpha^2}\,\lambda_1\phi(\lambda_1\sqrt{1-\alpha^2})\} \end{aligned} \tag{41}$$

$$\begin{aligned} e^*_{G^*} &= 1-(1-\alpha^2)\lambda_1^2\{(1-\alpha^2)\lambda_1^2 + [2\Phi(\lambda_1\sqrt{1-\alpha^2}) -1] - \\ &\quad - 2(1-\alpha^2)\lambda_1^2[1-\Phi(\lambda_1\sqrt{1-\alpha^2})] + 2\sqrt{1-\alpha^2}\,\lambda_1\phi(\lambda_1\sqrt{1-\alpha^2})\}^{-1} \end{aligned} \tag{42}$$

; where the expressions in (40), (41), and (42) provide respectively, the mean-squared error at the nominal, the influence function, and the breakdown point induced by the operation in (39), when the nominal process is as in (31).

The mean-squared error $e(\mu_o,G^*)$ in (40), and the breakdown point $e^*_{G^*}$ in (42) are both convex functions of λ_1. In Figure 1, we plot them against λ_1. In Figure 2, we plot the influence function $I_{G^*}(w)$ against $|w|$.

<u>Case m=2</u>

Then, from (37) and (34), and for λ_2 as in (38), we obtain:

$$\text{For } n\to\infty,\ G_n^*(y_1^{n-1}) = \alpha y_{n-1}\cdot\min\left(1,\lambda_2\{y_{n-2}^2 +(y_{n-1} - \alpha y_{n-2})^2(1-\alpha^2)\}^{-1/2}\right) \tag{43}$$

Then, we find,

$$e(\mu_o,G^*) = (1-\alpha^2)^{-1}\{1-\sqrt{2\pi}\,\alpha^2 \frac{\lambda_2}{\sqrt{1-\alpha^2}}[1-\Phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}})] + \sqrt{2\pi}\,\alpha^2[\phi(\frac{\lambda}{\sqrt{1-\alpha^2}}) - \phi(0)]\} \quad (44)$$

$$I_{G^*,2}(w) = \alpha^2 w^2 \cdot \min(1, \frac{\lambda_2^2}{w^2[1+(1-\alpha^2)(1-\alpha)^2]}) + \sqrt{2\pi}\,\alpha^2(1-\alpha^2)^{-1}\{\frac{\lambda_2}{\sqrt{1-\alpha^2}}[1-\Phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}})] + \phi(0) - \phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}})\} \quad (45)$$

$$e^*_{G^*,2} = 1-\alpha^2\lambda_2^2\{\alpha^2\lambda_2^2 + \sqrt{2\pi}\,\alpha^2[(1-\alpha^2)^{-1}+(1-\alpha)^2]\frac{\lambda_2}{\sqrt{1-\alpha^2}}[1-\Phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}})] + \sqrt{2\pi}\,\alpha^2[(1-\alpha^2)^{-1}+(1-\alpha)^2][\phi(0)-\phi(\frac{\lambda_2}{\sqrt{1-\alpha^2}})]\}^{-1} \quad (46)$$

In (45) and (46), size-two blocks of independent outlier vectors have been considered, as in (12) with m=2. As functions of λ_2, the mean-squared error in (44) and the breakdown point in (46) behave respectively as those in Figure 1. Also, the influence function in (45) behaves similarly to that in Figure 2. We note, that in the m=1 case, the found breakdown point and influence function are identical when size ℓ blocks of outlier vectors are considered, and for every $\ell \geq 1$.

Comparisons

Let us compare the operations derived for cases m=1 and m=2. Since the frequency of the outliers in a given system are at most approximately known, the thresholds λ_1 and λ_2 in the above operations are selected adhocly. Let us thus select:

$$\text{For } n\to\infty,\ G_n^*(y_1^{n-1}) = \begin{cases} \alpha y_{n-1} \min\left(1, \dfrac{\lambda}{\sqrt{1-\alpha^2}\,|y_{n-1}|}\right) ; \text{ for the m=1 case} \\ \\ \alpha y_{n-1} \min\left(1, \dfrac{\lambda}{\left\{(y_{n-1}-\alpha y_{n-2})^2 + \dfrac{y_{n-2}^2}{1-\alpha^2}\right\}^{1/2}}\right) ; \text{ for the m=2 case} \end{cases} \quad (47)$$

Let us denote by $I_m(w)$; m=1,2, the influence functions induced by the operations in (47), for the cases m=1 and m=2 respectively, when the nominal process is as in (31). Let us denote by e_m ; m=1,2, the corresponding mean-squared errors induced by the operations at the nominal in (31) process. Then, modifying the thresholds appropriately in expressions (40), (41), (44), and (45), we find after some tedious but straight forward manipulations:

$$e_1-e_2 = \alpha^2(1-\alpha^2)^{-1} F(\lambda) \quad (48)$$

$$I_1(w) - I_2(w) = -\alpha^2(1-\alpha^2)^{-1} F(\lambda) + \\ + \alpha^2 w^2\{\min(1, \frac{\lambda^2}{(1-\alpha^2)w^2}) - \min(1, \frac{\lambda^2}{[(1-\alpha^2)^{-1} + (1-\alpha)^2]w^2})\} \quad (49)$$

; where,

$$F(\lambda) \triangleq [2\lambda^2 + \sqrt{2\pi}\,\lambda + 2][1-\Phi(\lambda)] - [2\lambda + \sqrt{2\pi}]\phi(\lambda) \quad (50)$$

The function $F(\lambda)$ is nonpositive for all positive λ values, while the expression in the brackets of (49) is nonnegative for all λ and w. Thus,

$$e_2 \geq e_1 \text{ and } I_1(w) \geq I_2(w) \ ; \forall w, \forall\lambda \quad (51)$$

The inequalities in (51) express a tradeoff. Indeed, the operation for m=2 in (47) provides uniformly better protection against outliers than the operation for m=1, but the former induces a uniformly higher mean-squared error at the nominal process than the latter does. Generally, given each of the operations

separately, as λ increases, the mean-squared error at the nominal process decreases, but the breakdown point decreases as well, and the influence function increases uniformly. Thus, the selection of one operation among those in (47) and the choice of the threshold λ in it, depend on the desired tradeoff between performance at the nominal and protection against outliers.

V. CONCLUSIONS

We derived a class of outlier resistant prediction operations. Those operations are nonlinear functions of the observed data sequences and combine good performance in the absence of outliers with protection against data outliers. The class involves a threshold parameter and a data block size used as a basis in the construction. The two parameters are involved in a performance at the nominal process versus outlier resistance tradeoff. The selection of the threshold parameter is also based on a similar tradeoff. The operations in our class are qualitatively robust.

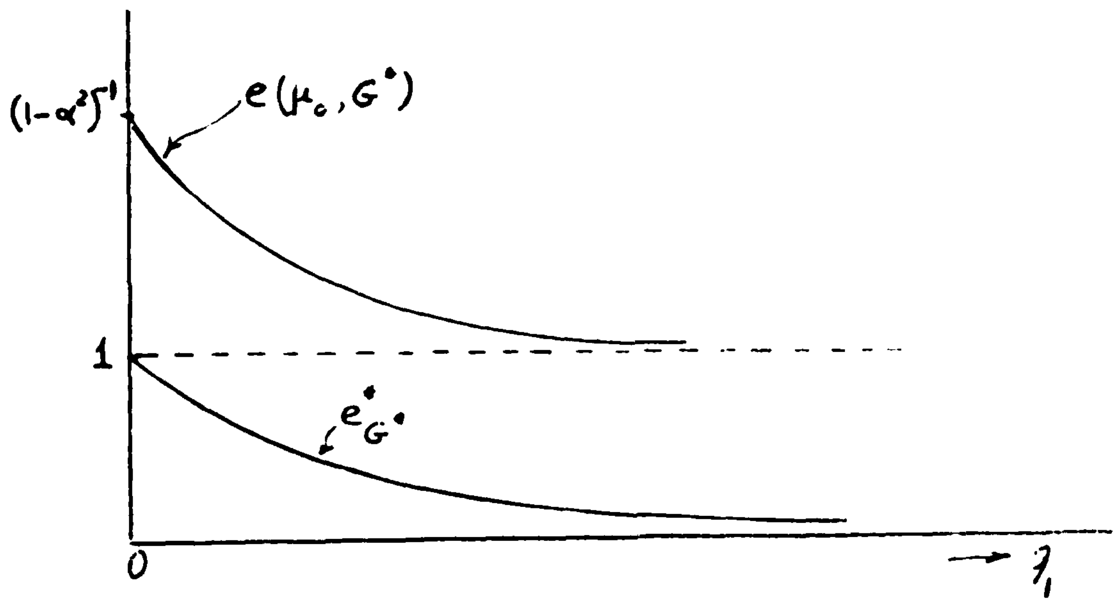

Figure 1

Autoregressive Gaussian Nominal Process and m=1

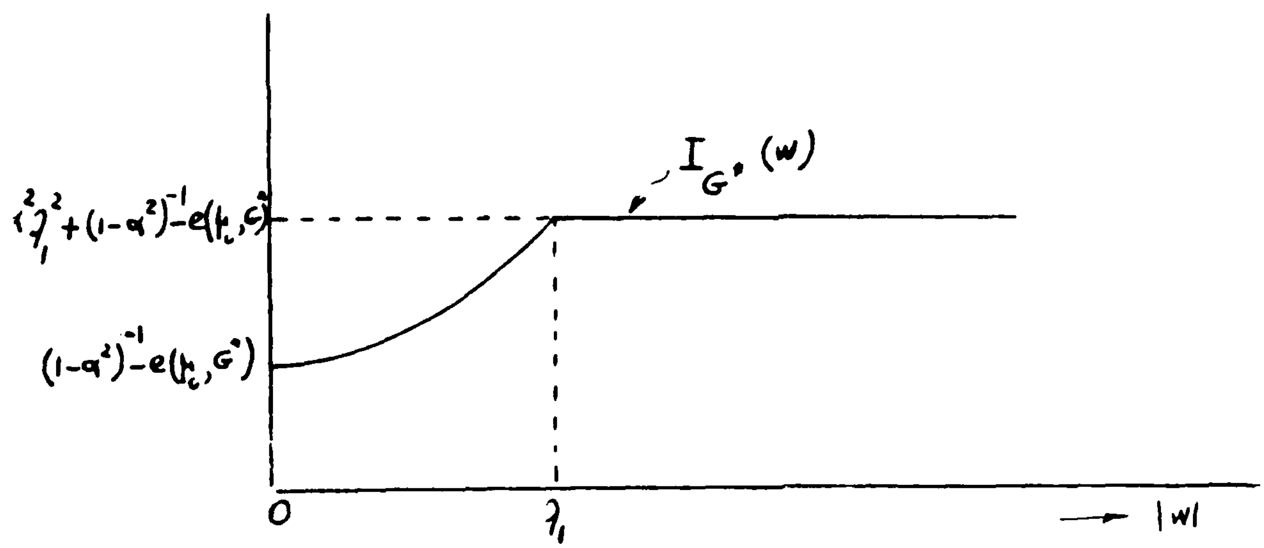

Figure 2

Autoregressive Gaussian Nominal Process and m=1

APPENDIX

Proof of Theorem 1

We easily find that,

$$\inf_{g_n} e_n(f,g_n) = E_{\mu_o}\{X_n^2\} - (1-\varepsilon)^2 \int_{R^{n-1}} f^{-1}(y_1^{n-1})[f_o(y_1^{n-1})m_o(y_1^{n-1})]^2 dy_1^{n-1}$$

So,

$\sup_{f\varepsilon F_n} \inf_{g_n} e_n(f,g_n)$ corresponds to:

$$\inf_{f\varepsilon F_n} \int_{R^{n-1}} f^{-1}(y_1^{n-1})[f_o(y_1^{n-1})m_o(y_1^{n-1})]^2 dy_1^{n-1} \qquad \text{(A.1)}$$

Applying calculus of variation on (A.1), subject to the constraints $\int_{R^{n-1}} f(y_1^{n-1})dy_1^{n-1} = 1$ and $f(y_1^{n-1}) - (1-\varepsilon)\ f_o(y_1^{n-1}) \geq 0;\ \forall\ y_1^{n-1}\varepsilon R^{n-1}$, we find the solution in the Theorem.

Proof of Theorem 2

Expression (29) is obvious, and is attained with equality iff $\lambda_n=0$ in (21). Regarding expression (30), applying the Schwartz inequality and using (22), we obtain,

$$\begin{aligned} e_n(\mu_o,G_n^*) &= E_{\mu_o}\{([X_n-m_o(Y_1^{n-1})]+[m_o(Y_1^{n-1})-G_n^*(Y_1^{n-1})])^2\} \\ &= E_{\mu_o}\{[X_n-m_o(Y_1^{n-1})]^2\} + E\{[m_o(Y_1^{n-1}) - G_n^*(Y_1^{n-1})]^2\} \\ &+ 2\ E_{\mu_o}\{[X_n-m_o(Y_1^{n-1})][m_o(Y_1^{n-1}) - G_n^*(Y_1^{n-1})]\} \qquad \text{(A.2)} \end{aligned}$$

; where,

$$|E_{\mu_o}\{[X_n - m_o(Y_1^{n-1})][m_o(Y_1^{n-1}) - G_n^*(Y_1^{n-1})]\}| \le$$

$$\le E_{\mu_o}^{1/2}\{[X_n - m_o(Y_1^{n-1})]^2\}\ E_{\mu_o}^{1/2}\{[m_o(Y_1^{n-1}) - G_n^*(Y_1^{n-1})]^2\} \tag{A.3}$$

From (A.2) and (A.3) we obtain:

$$|e_n^{1/2}(\mu_o, G_n^*) - E_{\mu_o}^{1/2}\{[X_n - m_o(Y_1^{n-1})]^2\}| \le E_{\mu_o}^{1/2}\{[m_o(Y_1^{n-1}) - G_n^*(Y_1^{n-1})]^2\} \tag{A.4}$$

Also,

$$E_{\mu_o}^{1/2}\{[m_o(Y_1^{n-1}) - G_n^*(Y_1^{n-1})]^2\} =$$

$$= E_{\mu_o}^{1/2}\left\{\left(\sum_{j=1}^{m} a_j^{(n-1)}\left[Y_j - \frac{g_{j+1}^*(Y_1^j) - g_{j+1}^*(0, Y_1^{j-1})}{a_j^{(j)}}\right]\right.\right.$$

$$\left.\left. + \sum_{j=m+1}^{n-1} a_j^{(n-1)}\left[Y_j - \frac{g_{m+1}^*(Y_{j-m+1}^j) - g^*(0, Y_{j-m+1}^{j-1})}{a_m^{(m)}}\right]\right)\right\} \le$$

$$\left|\sum_{j=1}^{m} a_j^{(n-1)} E_{\mu_o}^{1/2}\left\{\left[Y_j - \frac{g_{j+1}^*(Y_1^j) - g_{j+1}^*(0, Y_1^{j-1})}{a_j^{(j)}}\right]^2\right\} +\right.$$

$$\left. + \sum_{j=m+1}^{n-1} a_j^{(n-1)} E_{\mu_o}^{1/2}\left\{\left[Y_j - \frac{g_{m+1}^*(Y_{j-m+1}^j) - g_{m+1}^*(0, Y_{j-m+1}^{j-1})}{a_m^{(m)}}\right]^2\right\}\right| \le$$

$$\le D_m^* \sum_{j=m+1}^{n-1} |a_j^{(n-1)}| + \max_{1 \le j \le m} E_{\mu_o}^{1/2}\left\{\left[Y_j - \frac{g_{j+1}^*(Y_1^j) - g_{j+1}^*(0, Y_1^{j-1})}{a_j^{(j)}}\right]^2\right\} \sum_{j=1}^{m} \left|a_j^{(n-1)}\right| \tag{A.5}$$

$$\limsup_{n \to \infty} E_{\mu_o}^{1/2}\{[X_n - m_o(Y_1^{n-1})]^2\} = e_o^{1/2} \tag{A.6}$$

Applying (A.5) to (A.4), and taking limits, we obtain (30).

Proof of Theorem 3

Let us define,

$$a_i = \limsup_{k\to\infty} a_i^{(k)}$$

Then, we easily find,

$$I_{G^*}(w) = \lim_{\varepsilon\to 0} \frac{e(\mu_{\varepsilon,w},G^*) - e(\mu_o,G^*)}{\varepsilon} \leq$$

$$\leq -e(\mu_o,G^*) + E_{\mu_o}\{X^2\} + \left[\frac{2\lambda_m}{a_m^{(m)}}\right]^2 \sum_i a_i^2 +$$

$$+ 8\left[\frac{\lambda_m}{a_m^{(m)}}\right]^2 \sum_{i\neq j}\sum a_i a_j - 4\left[\frac{\lambda_m}{a_m^{(m)}}\right]^2 \sum_{i\neq j}\sum a_i a_j$$

$$\leq 4\left[\frac{\lambda_m}{a_m^{(m)}}\right]^2 (d^*)^2 + E_{\mu_o}\{X^2\} + e(\mu_o,G) < \infty,$$

since $e(\mu_o,G)$ is bounded via Theorem 2.

It can be easily found that $e(\mu_{\varepsilon,\infty},G^*)$ equals $e(\mu_o,G^*)$ at $\varepsilon=0$, and that it is monotonically increasing with increasing ε. In addition, $e(\mu_o,G^*)<E_{\mu_o}\{X^2\}$. Thus, the breakdown point $\varepsilon^*_{G^*}$ is positive.

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